Table 1 Coefficients $a^*_{j,k}$ of the series solution Eq. (22) for $R_T/Y_T=5$, K = 0.2

j^k	0	1	2	3	4	5	6	7	8	9
0	0.50000	1.0738	1.0938	1.1137	1.1336	1.1536	1.1735	1.1935	1.2134	1.2334
1	−0.56107	−1.6418	−3.2422	−5.3622	−8.0019	−11.161	−14.840	−19.039	−23.757	−28.995
2	−0.66527	−1.8458	−3.4329	−5.3180	−7.3925	−9.5478	−11.675	−13.666	−15.412	
3	−0.73217	−1.9496	−3.4151	−4.8599	−5.9834	−6.4533	−5.9058	−3.9455	−0.14486	
4	−0.77737	−1.9662	−3.1550	−3.8337	−3.3822	−1.0592	4.0098	12.843		
5	−0.80780	−1.9199	−2.6946	−2.2705	−0.47960	6.9999	19.108	39.058		
6	−0.82752	−1.8247	−2.0704	−0.23893	5.5236	17.750	39.829			
7	−0.83931	−1.6904	−1.3011	2.2441	11.789	31.452	67.079			
8	−0.84528	−1.5249	−0.39959	5.1906	19.436	48.775				
9	−0.84723	−1.3348	0.62406	8.6233	28.680	70.616				
10	−0.84680	−1.1267	1.7605	12.573	39.787					
11	−0.84557	−0.90747	2.9998	17.078	53.083					
12	−0.84514	−0.68450	4.3297	22.176						
13	−0.84721	−0.46630	5.7345	27.897						
14	−0.85360	−0.26262	7.1930							
15	−0.86631	−0.08518	8.6764							
16	−0.88754	0.05188								
17	−0.91976	0.13433								
18	−0.96574									
19	−1.0286									

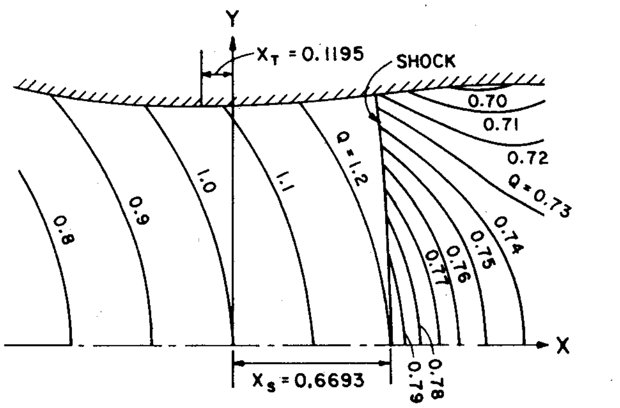

Fig. 3 Contours of constant speed and shape of nozzle after shock, for $R_T/Y_T=5$, $K=0.2$, $m=9$.

and a rough analysis suggests

$$h(\eta)=b_l(1-\eta^*)^2, \quad -b_l=\lim_{j\gg l}\frac{a_{j,0}}{a_{j-1,0}} \doteq \frac{a_{2m+1,0}}{a_{2m,0}} \tag{21}$$

The solution in the transformed variables is then written as [like Eq. (18)],

$$\phi=\sum_{j=0}^{2m+1}\sum_{k=0}^{m-[j/2]} a^*_{j,k}\xi^{*j}\eta^{*k} \tag{22}$$

The coefficients $a^*_{j,k}$ with $m=9$ are listed in Table 1.

That the transformations, Eqs. (19) and (20), considerably enlarge the convergent region is shown in Fig. 2. Note that the nozzle walls are at $\eta \doteq 1$. Between the nozzle walls and in this region, we have compared the results for the perturbation speed as the number of terms in the series is changed from $m=9$ to $m=8$. The discrepancy is found to be less than 0.5%.

Fig. 3 shows the contour lines of Q, $Q\equiv\sqrt{(a^*+\Phi_X)^2+\Phi_Y^2}$ calculated with Eq. (22) and Table 1, as well as the shape of the nozzle wall behind the shock, which is determined by numerical integration of Eq. (3) using the Runge-Kutta method. Conservatively, we limit our representation to $X<1.4$, where the convergence criterion $|\epsilon|=0.01$ is first met at the wall. Although by construction the wall slope is continuous, the curvature of the nozzle wall from the present solution is, as may be expected, discontinuous at the juncture of the wall and the shock. For the example shown, the radius of curvature is 6.337 just before the shock, and −4.861 just after the shock. This discontinuity of curvature arises from the requirements of a parabolic shock shape and an analytic solution behind the shock. Note that the special shock shape produces behind the shock a flow that decelerates at first, but soon accelerates locally near the wall.

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Mean Values of Unsteady Oscillations in Transonic Flow Calculations

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Introduction

In finite difference calculations of unsteady, oscillatory transonic flows about airfoils, it is observed that unsteady quantities oscillate about mean values which are different from their initial steady values. This is true of the shock positions on the upper and lower surfaces as well as the lift and moment coefficients. This is illustrated for the lift coefficient in Fig. 1.

The transonic indicial method[1,2] yields some insight into this phenomenon. It will be shown that a time constant characteristic of the indicial response fully accounts for this

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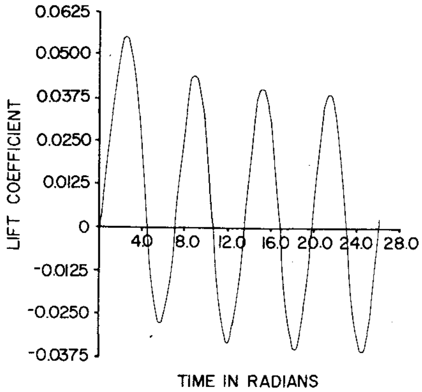

Fig. 1 LTRAN2 calculation of unsteady lift coefficients for NACA 64A006 airfoil oscillating in angle of attack. $\alpha(t)=0.25$ deg sin ωt; $k=0.2$.

discrepancy. An exponentially decaying transient solution is still present in most solutions after three or four cycles of oscillation; this causes a discrepancy of a few percent between the steady and mean unsteady quantities. With indicial data in hand it is possible either to check that enough cycles of the calculation have been run for the transient effects to die off, or, as in more expensive Euler equation calculations (e.g., Ref. 3), to correct the results after a few cycles to approximate better the limiting oscillations.

Analysis

The indicial response $A_\alpha(t)$ is the response in time of some aerodynamic quantity $A(t)$ to a unit step change in some flowfield parameter α at time zero. In the case to be studied here, A will represent the perturbations to the lift coefficients $[C_L - C_L \text{ (steady)}]$, and the upper and lower shock excursions $\delta x_s = [x_s - x_s\text{(steady)}]$. The steady values are subtracted off for mathematical convenience. The parameter α will be taken to be angle of attack. The indicial function must be calculated by finite differences, so it will include both physical and numerical effects. In the example below, both indicial and oscillatory calculations were done with the same code, LTRAN2 (Ref. 4), but the indicial analysis is independent of the code used.

When the indicial function $A_\alpha(t)$ is known, then the behavior of the quantity $A(t)$ can be calculated for an arbitrary schedule of parameter $\alpha(t)$ by means of Duhamel's convolution integral[5]:

$$A(t) = A_\alpha(t)\alpha(0) + \int_0^t A_\alpha(\tau)\frac{\mathrm{d}}{\mathrm{d}t}\alpha(t-\tau)\,\mathrm{d}\tau \tag{1}$$

For many cases of interest, the indicial response function closely approximates a decaying exponential with time constant β, that is,

$$A_\alpha(t) \simeq A_\infty(1-e^{-\beta t}) \tag{2}$$

Consider now a sinusoidal variation, $\alpha(t)=\alpha_0 \sin(\omega t)$, where $k=\omega c/U_\infty$ is the reduced frequency based on chord and α_0 is the amplitude expressed in units of the indicial step change. The unit of time is chosen so that a time lapse of 2π is equivalent to one chord length of freestream travel. Substituting Eq. (2) into Duhamel's integral for a sinusoidal variation in α results in an exponential integral which yields

$$A(t) = \frac{\alpha_0 A_\infty}{k^2+\beta^2}\left[\beta^2 \sin(kt) - k\beta\cos(kt) + k\beta e^{-\beta t}\right] \tag{3}$$

Table 1 Fitted values of β by number of time steps

Quantity	Number of steps 1200	240	150	Weighted value
δx_{sU}	.0160	.0217	.0213	.022
δx_{sL}	.0204	.0333	.0323	.033
C_L	.0171	.0287	.0330	.030

There is both an oscillatory part, with a phase lag, $\phi \approx \tan^{-1}(k/\beta)$, and a decaying exponential part. The average of A over the nth cycle of oscillation is defined by

$$\bar{A}_n \equiv \frac{k}{2\pi}\int_{t_{n-1}}^{t_n} A(\tau)\,\mathrm{d}\tau \qquad t_n \equiv \frac{2\pi n}{k} \tag{4}$$

For the result (3), one observes that oscillatory parts average out, leaving

$$\bar{A}_n = +\frac{A_\infty \alpha_0}{2\pi}\frac{k^2}{k^2+\beta^2}(e^{2\pi\beta/k}-1)e^{-2\pi\beta n/k} \tag{5}$$

The peak amplitude of oscillation is $\sqrt{2}$ times the root-mean-square value,

$$A_n \equiv \left(\frac{2}{\Delta t}\int_{t_{n-1}}^{t_n}(A(t)-\bar{A}_n)^2\mathrm{d}t\right)^{1/2} \qquad \Delta t \equiv \frac{2\pi}{k} \tag{6}$$

For the form of $A(t)$ given in Eq. (3), this works out to be

$$A_n = \frac{\alpha_0 A_\infty}{\sqrt{k^2+\beta^2}}\left[1-\left\{\frac{1}{4\pi}\frac{k^3}{k^2+\beta^2}(1-e^{4\pi\beta/k})e^{-(4\pi\beta n/k)}\right\}\right]^{1/2} \tag{7}$$

In the example that follows, the term in braces amounts to less than 1% of the final result even for $n=1$, but has been included for completeness. Finally, the ratio of mean value to peak value is

$$\frac{\bar{A}_n}{A_n} = \frac{1}{2\pi}\frac{k^2}{\sqrt{k^2+k^2}}(e^{2\pi\beta/k}-1)e^{-(2\pi\beta n/k)}[1-\{\ \}]^{-1/2} \tag{8}$$

The braces refer to the corresponding term in Eq. (7).

Example

We studied a NACA 64A006 airfoil at a freestream Mach number $M_\infty = 0.875$ oscillating in angle of attack about $\alpha = 0.0$ deg with an amplitude of 0.25 deg and reduced frequency based on chord $k=0.2$. The lift coefficient and upper and lower shock positions were computed using LTRAN2. Figure 1 shows the lift as a function of time; the variation of mean lift with time is apparent.

An LTRAN2 calculation of the indicial response to a 0.25 deg step change in angle of attack was run, as well as a steady, asymptotic solution at $\alpha=0.25$ deg. These results were then fitted to the exponential form of Eq. (2). Curve fits of the entire data set produced values which are too low to predict the observed oscillatory phenomena. This is because only the indicial data for the first 20% of the data set occur in Duhamel's integral, and moreover because the values for earlier times appear more often than those of later times. Therefore weighted curve fits were used which fit the first 20% of the indicial data set more closely. These values of β are given in Table 1. A more accurate fit could be obtained by

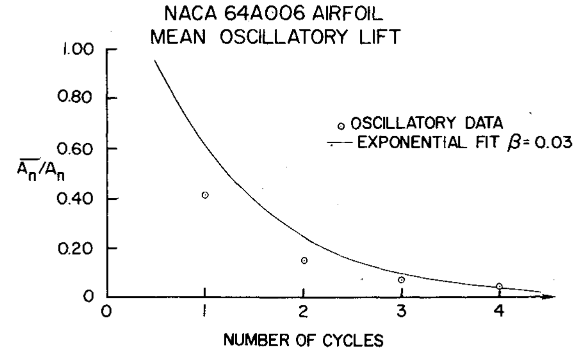

Fig. 2 Ratio of mean unsteady lift coefficient to oscillatory amplitude, NACA 64A006 airfoil.

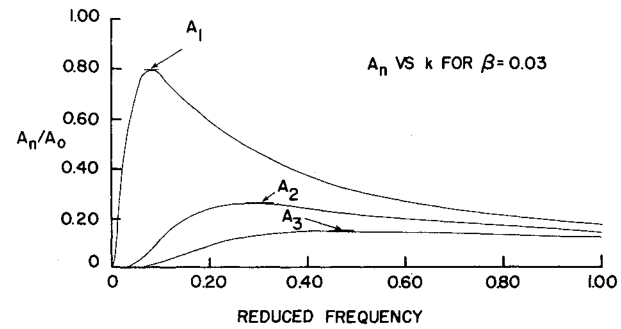

Fig. 3 Frequency dependence of mean unsteady lift coefficient for one, two, and three cycles of oscillation.

replacing Eq. (2) by a sum of exponentials, and by complicating the formalism slightly, but this will not be undertaken here.

With these values of β in hand, Eq. (8) was used to predict the mean unsteady amplitudes and peak amplitudes of C_L, the upper and lower shock excursions δx_{sU} and δx_{sL}, and their ratios. Note that $\alpha_0 = 1$ here, being the ratio of oscillatory amplitude in angle of attack to the indicial step. These predictions for lift, as a function of n, are compared with the results calculated directly from oscillatory data in Fig. 2. There is good agreement for n greater than 1. Even for $n = 4$, the mean value of lift is still a few percent of the oscillatory amplitude. According to Eq. (8), we would have to wait at least six cycles before the discrepancy falls to a level of 1%.

Alternatively, one could use Eq. (8) to correct the results for, say, $n = 4$ to approximate the results for $n \to \infty$.

Another important aspect of this result is shown in Fig. 3, where A_n is plotted as a function of reduced frequency for $n = 1$, 2, and 3. For the value $\beta = 0.03$ for lift, we find that A_1 rises rapidly from zero to a maximum 79% at $k = 0.08$. Similarly, A_2 has its maximum at $k = 0.28$ and A_3 at $k = 0.47$. Significantly, these frequencies are in the middle of the range of applicability of unsteady transonic finite difference codes.

Conclusion

Analysis of indicial data for unsteady transonic flows predicts the discrepancy between the mean oscillatory aerodynamic forces and the corresponding steady values, as well as the dependence of this discrepancy on reduced frequency and number of cycles run. A simple formula relates the time constant of the indicial response to the number of cycles of calculation needed to allow transient effects to die out.

Added note: A similar investigation by Seebass and Fung[6] has been brought to our attention.

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Digital Data Reduction of Oscillatory Signals

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Introduction

OFTEN, one is interested in the history of the amplitudes and frequencies that occur in measured signals. In case of oscillatory combustion in solid propellant rocket motors, typical propellant characteristics, such as the solid propellant response function, are derived from the amplitude growth of the pressure oscillations. Another example is the POGO-behavior of liquid propellant rocket motors, where the frequency spectrum yields important information about the propellant feedline dynamics. In order to reduce the measured data involving oscillatory signals, a Fourier analysis may be applied. However, if the signal contains oscillatory components with varying amplitudes and frequencies, Fourier analyses may become rather time-consuming. In that case, each oscillation has to be analyzed separately.

In addition, noise sometimes hampers a Fourier analysis. Usually, noise-containing signals are filtered electronically before analyzing the data, but this may cause a loss of information.

An alternative approach is to apply a purely digital recognition technique. Such a digital data reduction process has been developed recently[1] and is described here. This technique has been applied successfully to reduce data obtained during oscillatory combustion.[2] The basic approach is to select those extremes out of a series of digitized data that are considered to yield significant information about the frequency and amplitude histories.

Preliminary Data Processing

In the case of an analog signal, the signal is digitized at a sample frequency at least one order of magnitude larger than the highest frequency of interest. From these digitized data only the local extremes are selected by means of the following procedure:

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